Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Extension of the λ Formulation to Imperfect Gas Flows"

G. Christy* and G. Emanuel†
University of Oklahoma, Norman, Oklahoma 73019

Introduction

AN efficient method for solving the unsteady Euler equations is the λ scheme.[2] It is a second-order accurate technique that utilizes Riemann variables for obtaining a solution, including a steady one. It is particularly suitable for transonic flows and flows with shock waves. Lentini[1] extends this formulation to an imperfect gas flow. As an illustrative example, he applies it to a steady, quasi-one-dimensional air flow through a converging/diverging nozzle. The stagnation temperature is 2000 K, and because of the equilibrium assumption for the vibrational modes of O_2 and N_2, the specific heats are temperature dependent. NASA[3] thermochemical polynomials are used for the O_2 and N_2 species. Results are compared with a perfect gas calculation with $\gamma = 1.4$ for the ratio of specific heats. A mass flow rate comparison yields an overestimate of 4.1% for a perfect gas relative to the imperfect gas result. As Lentini[1] notes, this is a large difference; it has an impact on the thrust estimation.

Discussion

An analogous mass flow rate comparison is performed in Ref. 4, where the comparable mass flow rate overestimate is only 2.5%. Reference 4 utilizes a standard equilibrium or frozen computation between the plenum and throat, i.e., no λ scheme, and a thermally perfect gas, as does Lentini, but with a harmonic oscillator assumption for a single diatomic species. With a characteristic vibrational temperature of 3056 K, this model is applicable to air[5] for the temperature range being discussed.

The decrease in temperature from the stagnation state to the sonic throat is only about 200 K. Over this limited temperature range, there is only a small change in γ. Hence, an isentropic mass flow rate calculation, with γ evaluated at an average temperature of 1900 K should verify the cited values. The harmonic oscillator calculation yields a 2.6% overestimate in accord with the Ref. 4 case. On the other hand, the NASA[3] polynomials for the O_2 and N_2 specific heats at constant pressure yields a 3.5% overestimate, still well below the 4.1% value. This comparison, to some extent, shows the sensitivity of the mass flow rate to the choice of a thermodynamic model, even though the change in state between plenum and throat is not large. It also suggests a re-examination of the accuracy of the λ scheme for this type of calculation.

Received July 15, 1993; revision received Aug. 16, 1993; accepted for publication Aug. 19, 1993.

*Graduate Student, School of Aerospace and Mechanical Engineering.
†Professor, School of Aerospace and Mechanical Engineering. Associate Fellow AIAA.

References

[1]Lentini, D., "Extension of the Formulation to Imperfect Gas Flows," *AIAA Journal*, Vol. 30, No. 11, 1992, pp. 2785–2788.
[2]Moretti, G., "The λ-Scheme," *Computers and Fluids*, Vol. 7, No. 3, 1979, pp. 191–205.
[3]Gardiner, W. C., (ed.), "Table of Coefficient Sets for NASA Polynomials," *Combustion Chemistry*, Springer-Verlag, New York, 1984, pp. 485–504.
[4]Christy, G., "Calorically Imperfect Isentropic Flow," M.S. Thesis, University of Oklahoma, 1993.
[5]Ames Research Staff, "Equations, Tables, and Charts for Compressible Flow," Moffett Field, CA, 1953, p. 15.

Reply by the Author to G. Christy and G. Emanuel‡

Diego Lentini§
Università degli Studi di Roma "La Sapienza," I-00184 Rome, Italy

Discussion

THE NASA polynomials for species O_2, as reported in Ref. 3 of the Comment, feature an error (a missing minus sign) in the third coefficient (i.e., a_3 for $T > 1000$ K). This accounts for the discrepancy observed with respect to results computed by Christy and Emanuel by using the harmonic oscillator. The model described in Ref. 1 leads to virtually the same results obtained with the harmonic oscillator when the NASA polynomials are used with the correct signs. Unfortunately, in Ref. 1 the functions defined by Eqs. (13–17) were interpolated in the whole temperature range 200–2000 K by means of fourth-order orthogonal polynomials, whereas we later found it necessary to use sixth-order to get an accurate solution (alternatively, one may resort to table look-up). The corrected results for imperfect gas flow, when compared to perfect gas results (with $\gamma = 1.4$), indicate that the latter overpredict the mass flow rate by 2.6%, and underpredict the velocity thrust by 1.2%; accordingly, the specific impulse is underestimated by as much as 3.7%. The temperature discrepancy between the two models at the nozzle exit is still about 150 K. Incidentally, the results obtained using 40 computational nodes are grid-independent.

Reference

[1]Lentini, D., "Extension of the Formulation to Imperfect Gas Flows," *AIAA Journal*, Vol. 30, No. 11, 1992, pp. 2785–2788.

‡Received Oct. 27, 1993; revision received Nov. 2, 1993; accepted for publication Nov. 2, 1993.

§Researcher, Dipartimento di Meccanica e Aeronautica. Member AIAA.